

Quark-Antiquark Condensates in the Hadronic Phase

A. Tawfik¹ and D. Toublan²

¹*Faculty of Physics, University of Bielefeld, D-33501 Bielefeld, Germany*

²*Physics Department, University of Illinois at Urbana-Champaign, Urbana, IL 61801*

(Dated: 2nd February 2008)

We use a hadron resonance gas model to calculate the quark-antiquark condensates for light (up and down) and strange quark flavors at finite temperatures and chemical potentials. At zero chemical potentials, we find that at the temperature where the light quark-antiquark condensates entirely vanish the strange quark-antiquark condensate still keeps a relatively large fraction of its value in the vacuum. This is in agreement with results obtained in lattice simulations and in chiral perturbation theory at finite temperature and zero chemical potentials. Furthermore, we find that this effect slowly disappears at larger baryon chemical potential. These results might have significant consequences for our understanding of QCD at finite temperatures and chemical potentials. Concretely, our results imply that there might be a domain of temperatures where chiral symmetry is restored for light quarks, but still broken for strange quark that persists at small chemical potentials. This might have practical consequences for heavy ion collision experiments.

I. INTRODUCTION

There are many important physical systems where the strong interaction at nonzero temperature and density plays a crucial role: e.g., neutron stars, heavy-ion collision experiments, and the early universe. To understand these systems, we need to better grasp the phase diagram of QCD at nonzero temperature and chemical potentials. Unfortunately, the most reliable techniques used at nonzero temperature and zero chemical potentials are generally not applicable at finite chemical potentials. In particular, lattice simulations suffer from the so-called sign problem. This problem has not yet been solved explicitly. However, some recent advances have allowed the study of the high temperature and low chemical potential part of the phase diagram [1, 2, 3, 4, 5, 6]. Our current understanding of the phase diagram is therefore mainly based on various models which validity is difficult to test [7]. In this article we shall use a hadron resonance gas model to study properties of the QCD phase diagram at nonzero temperature and chemical potentials. It is one of the models that has been successfully used in the past to study the hadronic phase at finite temperature and chemical potentials [8, 9, 10, 11]. This model is in good quantitative agreement with the lattice simulations for various observables in the hadronic phase [10]. Furthermore, it has also been used to determine the critical temperature that separates the hadronic phase from the quark-gluon plasma phase [11]. We would like to stress here again that the agreement with the lattice simulations is excellent, especially at small chemical potential [1, 2, 3, 4].

Recent lattice simulations at finite temperatures and zero chemical potentials seem to indicate that the critical temperature for the light quark-antiquark condensate is different than that for the strange quark-antiquark condensate [12]. In particular, the lattice results for the different quark number susceptibilities indicate that the pseudocritical temperature of the crossover for the restoration of chiral symmetry for the light quarks is smaller than that for the strange quark [12]. This difference has also been observed in chiral perturbation theory at zero chemical potential [13]. In this article we shall use the hadron resonance gas model to study the possible differences between the behavior of the light $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ and strange $\langle \bar{s}s \rangle$ quark-antiquark condensates in the hadronic phase. At small chemical potentials, we find that the strange quark-antiquark condensate is still large where the light quark-antiquark condensates become very small. At higher chemical potentials, this difference slowly diminishes: The light and strange quark-antiquark condensates become small at the same temperature. The difference between the light and strange quark-antiquark condensates might lead to significant consequences for the heavy-ion collision experiments. One can expect the existence of a domain of temperatures where the chiral symmetry is restored for the light quark flavors and broken for the strange quark flavor.

II. FORMALISM

The pressure in the hadronic phase is given by the contributions of all the hadron resonances up to 2 GeV treated as a free gas [10, 11]. All thermodynamic quantities can be derived from the pressure

$$p = \lim_{V \rightarrow \infty} \frac{T}{V} \ln Z(T, \mu_B, \mu_I, \mu_S, V), \quad (1)$$

where $Z(T, \mu_B, \mu_I, \mu_S, V)$ is the grand canonical partition function in a finite volume V , at nonzero temperature, T , baryon chemical potential, μ_B , isospin chemical potential, μ_I , and strangeness chemical potential, μ_S . It has

been found that this model gives a very good description of the hadronic phase and of the critical temperature that separates the hadronic phase from the quark-gluon plasma phase [10, 11].

In the free gas approximation, the contribution to the pressure due to a particle of mass m_h , baryon charge B , isospin I_3 , strangeness S , and degeneracy g is given by

$$\Delta p = \frac{g m_h^2 T^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n^2} \exp\left(n \frac{B\mu_B - I_3\mu_I - S\mu_S}{T}\right) K_2\left(n \frac{m_h}{T}\right), \quad (2)$$

where $\eta = +1$ for fermions and $\eta = -1$ for bosons. $K_n(x)$ is the modified Bessel function. In the hadronic phase the isospin is an almost exact symmetry since the hadronic spectrum is almost isospin symmetric [11].

The quark-antiquark condensates are given by the derivative of the pressure with respect to the constituent quark masses:

$$\begin{aligned} \langle \bar{q}q \rangle &= \langle \bar{q}q \rangle_0 + \sum_h \frac{\partial m_h}{\partial m_q} \frac{\partial \Delta p}{\partial m_h}, \\ \langle \bar{s}s \rangle &= \langle \bar{s}s \rangle_0 + \sum_h \frac{\partial m_h}{\partial m_s} \frac{\partial \Delta p}{\partial m_h}, \end{aligned} \quad (3)$$

where $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ represents the light quark-antiquark condensate. $\langle \bar{q}q \rangle_0$ and $\langle \bar{s}s \rangle_0$ indicate the value of the light and strange quark-antiquark condensates in the vacuum, respectively.

The computation of the quark-antiquark condensates in Eq. (3) requires modeling two quantities. The first one is the strange quark-antiquark condensate at zero temperature and zero chemical potential, $\langle \bar{s}s \rangle_0$. For it we need to know the kaon decay constant, which is unfortunately not yet known experimentally. But in the framework of lattice QCD, the ratio of heavy to light meson decay constants is found to be given by $F_K/F_\pi = 1.16 \pm 0.04$ [14] (see also [15]). Using QCD sum rules, the ratio of strange to light quark-antiquark condensates in vacuum is given by 0.8 ± 0.3 [16] (another estimation gives 0.75 ± 0.12 [17]). The Gell-Mann–Oakes–Renner relation connects the quark-antiquark condensates at zero temperature and zero chemical potential to the meson masses and to their decay constants. We use the next-to-leading order result obtained in chiral perturbation theory [18]:

$$\begin{aligned} F_\pi^2 m_\pi^2 \left(1 - \kappa \frac{m_\pi^2}{F_\pi^2}\right) &= (m_u + m_d) \langle \bar{q}q \rangle_0, \\ F_K^2 m_K^2 \left(1 - \kappa \frac{m_K^2}{F_\pi^2}\right) &= \frac{1}{2}(m_q + m_s)(\langle \bar{q}q \rangle_0 + \langle \bar{s}s \rangle_0), \end{aligned} \quad (4)$$

where m_q stands for the light quark mass where we assume $m_u = m_d$. The coefficient $\kappa = 0.021 \pm 0.008$ has been obtained from the low-energy coupling constants of chiral perturbation theory [16, 18]. Notice that κ does not contain any chiral logarithm.

The second quantity which we have to model in order to calculate the quark-antiquark condensates in the hadron gas model Eq. (3) is the quark mass dependence of the hadron masses. Results from lattice simulations [10] indicate that

$$\frac{\partial m_h}{\partial (m_\pi^2)} = \frac{A}{m_h}, \quad (5)$$

where $A \sim 0.9 - 1.2$. Using these results together with the Gell-Mann–Oakes–Renner relation, Eq. (4), one finds that [11]

$$\frac{\partial m_h}{\partial m_q} \equiv \frac{\partial (m_\pi^2)}{\partial m_q} \frac{\partial m_h}{\partial (m_\pi^2)} = \frac{A \langle \bar{q}q \rangle_0 \left(1 + 2\kappa \frac{m_\pi^2}{F_\pi^2}\right)}{F_\pi^2 m_h}. \quad (6)$$

$$\frac{\partial m_h}{\partial m_s} \equiv \frac{\partial (m_K^2)}{\partial m_s} \frac{\partial (m_\pi^2)}{\partial (m_K^2)} \frac{\partial m_h}{\partial (m_\pi^2)} = \frac{A \langle \bar{q}q \rangle_0 \left(1 + 2\kappa \frac{m_\pi^2}{F_\pi^2}\right)}{F_\pi^2 m_h}. \quad (7)$$

Relation (7) is not valid for the pion mass, since pions are almost independent of m_s in chiral perturbation theory [18]. However, relations (6) and (7) work reasonably well for the nucleon [19], for instance. We therefrom assume that this relation is valid for all the hadrons heavier than the pions. This is in agreement with the lattice simulations, where the masses of a few hadrons have been shown to follow Eq. (6) and Eq. (7) over a sizable range of quark masses [10].

Using these assumptions, we find that the contribution of one hadron of mass m_h to the light quark-antiquark condensates is given by

$$\frac{\Delta \langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} = -\frac{g}{2\pi^2} T m_h \frac{A(1+2\kappa \frac{m_\pi^2}{F_\pi^2})}{F_\pi^2} \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n} \exp\left(n \frac{B\mu_B - I_3\mu_I - S\mu_S}{T}\right) K_1\left(n \frac{m_h}{T}\right), \quad (8)$$

whereas its contribution to the strange quark-antiquark condensate is given by

$$\frac{\Delta \langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0} = -\frac{g}{2\pi^2} T m_h \frac{A(1+2\kappa \frac{m_\pi^2}{F_\pi^2})}{F_\pi^2} \frac{\langle \bar{q}q \rangle_0}{\langle \bar{s}s \rangle_0} \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n} \exp\left(n \frac{B\mu_B - I_3\mu_I - S\mu_S}{T}\right) K_1\left(n \frac{m_h}{T}\right). \quad (9)$$

III. RESULTS

We include all hadron resonances with masses up to ~ 2 GeV. The light and strange quark-antiquark condensates, Eq. (8) and Eq. (9) are calculated as a function of temperature T and for various values of baryon, isospin, and strangeness chemical potentials, μ_B , μ_I , and μ_D , respectively. The results are shown in Fig. 1 and Fig. 2.

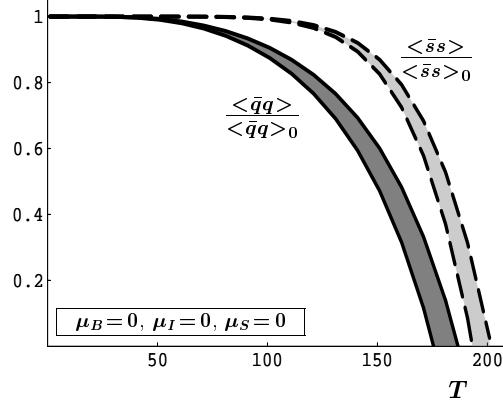


Fig. 1: The quark-antiquark condensates as a function of temperature T at zero baryon, isospin, and strangeness chemical potentials. The light quark-antiquark condensate $\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0}$ is in dark gray and solid curve. The strange quark-antiquark condensate $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0}$ is given in light gray and dashed curve.

At zero chemical potential, we find that the strange quark-antiquark condensate is still large at temperatures where the light quark-antiquark condensates become small: $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0} = 0.4 \pm 0.2$ where $\langle \bar{q}q \rangle$ vanishes. As can be seen in Fig. 2, an increase in μ_B tends to reduce the difference between $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$. Increasing μ_I apparently accentuates this difference. We also notice that increasing μ_S does not sizably affect. The reason for this behavior is that the light quark-antiquark condensates in the hadron resonance gas model are much more sensitive to the pion physics than the strange quark-antiquark one. On the other hand, we find that all condensates are equally sensitive to μ_B . The value of $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0}$ at the critical temperature for the light quark flavors is shown in Fig. 3 as a function of μ_B . We study two cases: $\mu_I = 0$ and $\mu_S = 0$ and $\mu_I = 0$ and $\mu_S = \mu_B/3$. The latter is very much compatible with the corresponding values calculated for recent heavy-ion collision experiments, as mentioned above.

We conclude that the strange quark-antiquark condensate can be relatively large at temperatures where the light quark-antiquark condensate is very small. We obtained these results from a hadron resonance gas model. Our results agree with the recent MILC lattice simulations at zero temperature and chemical potentials [12]. The uncertainty of the value of the strange quark-antiquark condensate at this critical temperature is however rather large. This is due to the uncertainty in the values of the condensates at high temperatures which we have calculated by the hadron resonance gas model. This uncertainty could be reduced for the most part, if the factor A in Eq. (5) were more precisely known.

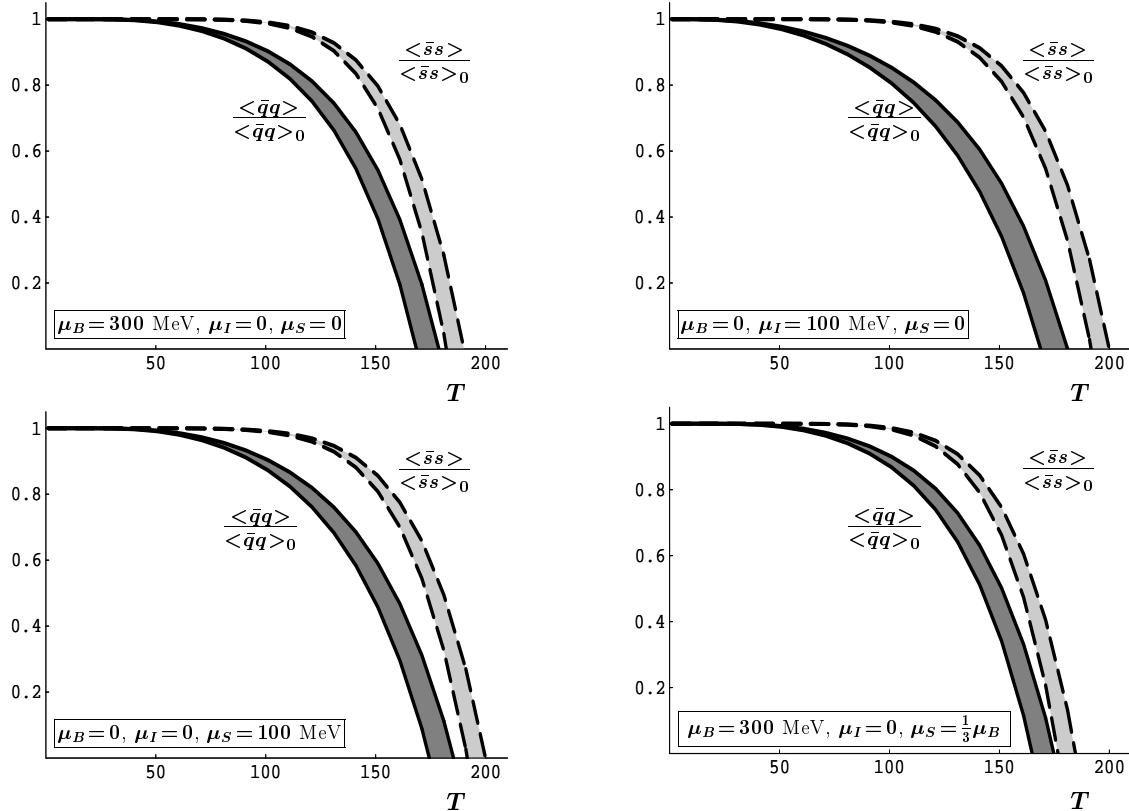


Fig. 2: The quark-antiquark condensates as a function of temperature at different values of baryon, isospin, and strangeness chemical potentials. The light quark-antiquark condensate $\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0}$ is in dark gray and solid curve. The strange quark-antiquark condensate $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0}$ is given in light gray and dashed curve. Upper-left panel: $\mu_B = 300$ MeV, $\mu_I = 0$, and $\mu_S = 0$. Upper-right panel: $\mu_B = 0$, $\mu_I = 100$ MeV, and $\mu_S = 0$. Lower-left panel: $\mu_B = 0$, $\mu_I = 0$, and $\mu_S = 100$ MeV. Lower-right panel: $\mu_B = 300$ MeV, $\mu_I = 0$, and $\mu_S = 100$ MeV. Although chemical potentials have been set to finite values, the critical temperature of light quark condensate is still smaller than that of the strange quark one.

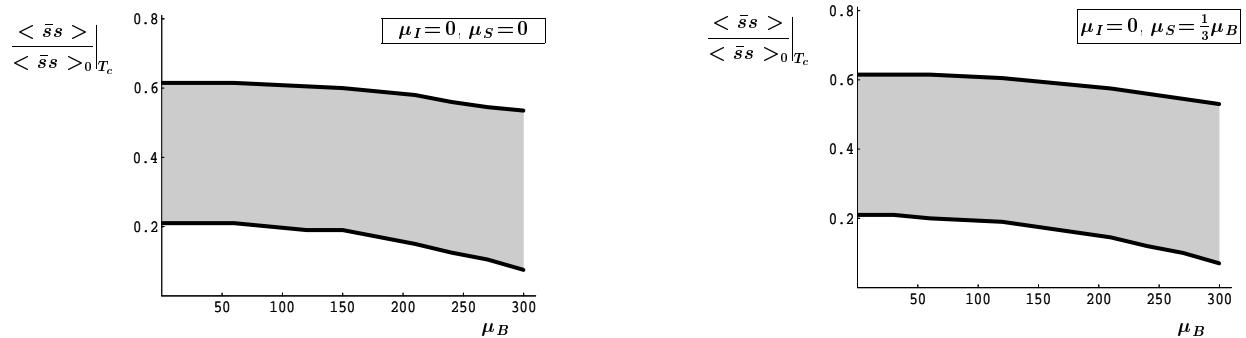


Fig. 3: The value of $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0}$ at the temperature where $\langle \bar{q}q \rangle$ vanishes given as a function of baryon chemical potential μ_B . On left panel, we assign μ_I and μ_S to zero. On right panel, we set $\mu_I = 0$ and $\mu_S = \frac{1}{3}\mu_B$. Latter values are relevant to the values measured in recent heavy-ion collisions.

IV. CONCLUSION

We have shown that in the hadron resonance gas model, the strange quark-antiquark condensate can still be sizably large at temperatures where the light quark condensates nearly vanish. In other words, the hadron resonance gas model seems to indicate that the chiral symmetry could still be broken for the strange quark flavor at temperatures where chiral symmetry is restored for the light quark flavors. This new situation seems to be possible only in a range of a few

tens of MeV around $T = 180$ MeV. This result is in agreement with recent lattice calculation at zero chemical potential. We have shown that this effect persists for small chemical potentials and that it seems to slowly disappear at higher chemical potentials. We have also shown that this effect can be enhanced by a nonzero isospin chemical potential. This fact could be used in heavy-ion collision experiments by choosing different isotopes, which is an interesting option for other reasons as well [20].

This effect does not contradict any fundamental principle. At nonzero quark masses, lattice simulations show that in QCD the hadronic phase and the quark-gluon plasma phase are separated by a crossover, not by a sharp phase transition. It is therefore possible for the light quark-antiquark condensates to be small while the strange quark-antiquark condensate is large in the crossover region. However, it is generally expected that this crossover stops and becomes a first order phase transition with increasing μ_B . In this latter case, $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$ should drop and become small at the same temperature. Furthermore, the discontinuity of $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$ at a first order phase transition are related by a Clausius-Clapeyron relation: They should be similar [21, 22]. The emergence of a first order phase transition at nonzero μ_B is therefore entirely possible in the hadron resonance gas model, since the difference between $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$ is smaller at larger μ_B . Notice however, that the difference between $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$ is larger for larger μ_I . This would tend to indicate that there is no critical endpoint in the QCD phase diagram at nonzero temperature and isospin chemical potential.

Finally, we observe that if this effect is true, our study opens the possibility of the existence of a domain where chiral symmetry is partially restored with a spontaneous chiral symmetry breaking given by $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \times SU(2)_A$. It would be interesting to study the physics of such a spontaneous symmetry breaking of chiral symmetry and determine its consequences for heavy-ion collision experiments.

[1] Z. Fodor and S. D. Katz, Phys. Lett. B **534**, 87 (2002); JHEP **0203**, 014 (2002); JHEP **0404**, 050 (2004);
[2] C. R. Allton *et al.*, Phys. Rev. D **66**, 074507 (2002); Phys. Rev. D **68**, 014507 (2003); F. Karsch *et al.*, Nucl. Phys. Proc. Suppl. **129**, 614 (2004); C. R. Allton *et al.*, Phys. Rev. D **71**, 054508 (2005).
[3] P. de Forcrand and O. Philipsen, Nucl. Phys. B **642**, 290 (2002); Nucl. Phys. B **673**, 170 (2003); Nucl. Phys. Proc. Suppl. **129**, 521 (2004).
[4] M. D'Elia and M. P. Lombardo, Phys. Rev. D **67**, 014505 (2003).
[5] R. V. Gavai and S. Gupta, Phys. Rev. D **68**, 034506 (2003); R. Gavai, S. Gupta and R. Roy, Prog. Theor. Phys. Suppl. **153**, 270 (2004).
[6] V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, arXiv:hep-lat/0503010.
[7] For a review see, K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333;
[8] For a review see, P. Braun-Munzinger, K. Redlich and J. Stachel, arXiv:nucl-th/0304013.
[9] J. Cleymans and K. Redlich, Phys. Rev. Lett. **81**, 5284 (1998).
[10] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C **29**, 549 (2003); Phys. Lett. B **571**, 67 (2003); F. Karsch, Prog. Theor. Phys. Suppl. **153**, 106 (2004); K. Redlich, F. Karsch and A. Tawfik, J. Phys. G **30**, S1271 (2004), M. D'Elia and M. P. Lombardo, arXiv:hep-lat/0406012; A. Tawfik, hep-ph/0410329, A. Tawfik, Phys. Rev. D **71**, 054502 (2005);
[11] D. Toublan and J. B. Kogut, Phys. Lett. B **605**, 129 (2005).
[12] C. Bernard *et al.* [MILC Collaboration], Phys. Rev. D **71**, 034504 (2005).
[13] J. R. Pelaez, Phys. Rev. D **66**, 096007 (2002).
[14] C. W. Bernard, Nucl. Phys. Proc. Suppl. **94**, 159 (2001) [arXiv:hep-lat/0011064].
[15] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
[16] M. Jamin, Phys. Lett. B **538**, 71 (2002) [arXiv:hep-ph/0201174].
[17] S. Narison, arXiv:hep-ph/0202200.
[18] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985).
[19] M. E. Sainio, PiN Newslett. **16**, 138 (2002).
[20] B. Klein, D. Toublan and J. J. M. Verbaarschot, Phys. Rev. D **68**, 014009 (2003); D. Toublan and J. B. Kogut, Phys. Lett. B **564**, 212 (2003); M. Frank, M. Buballa and M. Oertel, Phys. Lett. B **562**, 221 (2003); A. Barducci, R. Casalbuoni, G. Pettini and L. Ravagli, Phys. Rev. D **69**, 096004 (2004). A. Barducci, G. Pettini, L. Ravagli and R. Casalbuoni, Phys. Lett. B **564**, 217 (2003).
[21] H. Leutwyler, Phys. Lett. B **284**, 106 (1992); A. Barducci, R. Casalbuoni, G. Pettini and R. Gatto, Phys. Lett. B **301**, 95 (1993);
[22] D. Toublan, arXiv:hep-th/0501069.